

On Portfolio Selection when Asset Returns are Elliptical.

Mohamed TOUATI-TLIBA¹

Abstract

For many markets, returns have fatter than normal tails with an extremely high sample kurtosis. Hence the class of elliptical distributions provides attractive and appealing alternatives to normal distributions in modeling the empirical distribution of returns. We derive a result related to the covariance of functions of elliptical random variables. Based on this result, we obtain the conditions of optimality for the portfolio choice problem and show how to derive many useful results for the elliptical class of distributions including some main results in the studies of Chamberlain and of Owen and Rabinovitch. In particular, we obtain the efficient set, in the portfolio space, analytically. Furthermore, we derive a general global risk aversion measure, relevant to the case of elliptical risk, which generalizes the Rubinstein measure. We discuss, in the context of the portfolio choice problem, differences between normal and non normal elliptical distributions, especially those with heavier tails than normal, focusing on the role of Kurtosis as a measure of fatness of tails in determining the optimal investment strategy of risk-averse investors. If a riskless asset exists, the sensitivity of expected utility to kurtosis implies that a risk-averse investor demand for risk is smaller when faced with a fat tails elliptical distribution instead of a normal distribution that presents the same mean-variance choices. When the kurtosis measure gets sufficiently large, a risk-averse investor tends to invest all his wealth in the riskless asset even when the variance remains constant. Finally, we show that short sales are not an optimal investment strategy for all risk-averse investors if and only if the means of asset returns are equal and the inverse of the variance-covariance matrix has non negative (positive) row sums.

Key words: Portfolio Selection; Elliptical Distribution; Scale Mixture of Normals; Kurtosis; Global Measure of Risk-Aversion; Stochastic Dominance.

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